ME230 Lecture Notes

DC Motors and PID Control

1 Introduction to Control Systems

The purpose of a control system is to achieve the desired output from a physical system by controlling the input to a system. The components of a control system include sensors which measure system output, actuators which generate the input to the system, and the controller which is responsible for deciding what input will be sent to the system based upon the measured output.

In this handout the topic of actuators and controllers will be addressed. In particular, we will discuss permanent magnet DC brush motors and PID controllers.

2 Permanent Magnet DC Brush Motors

Motors are the primary method for actuation in a mechatronic system. Of the many types of motors available, the permanent magnet DC brush motor is the most basic, and are the type used in the lab. An example of this type of motor is given in Figure 1.

2.1 Anatomy of a Permanent Magnet DC Brush Motor

The motor can be divided into two main elements — the stator (stationary component) and the rotor (rotational component). The stator resides on the outer part of the motor and is composed of permanent magnets that have the form of a hollow cylinder. Figure 2 shows a stator which uses two permanent magnets. The purpose of the stator is to generate a constant magnetic field between the pieces. Note the gap between the magnets.

The armature windings shown in Figure 3 are formed by highly elongated ovals of wire wound around deep grooves running down the length of the iron cylindrical rotor. When current flows through a winding that's near one of the gaps between the magnets, it produces a torque on the rotor that tends to turn it. This is caused due to the fact that a force is applied to a current carrying conductor in a magnetic field. To continue rotation of the motor, different armature windings must be energized. This process is taken care of through the use of brushes and commutator rings. Figure 4 is a simplified diagram of this operation.

Figure 1: Permanent magnet DC brush motor. Note the voltage terminals and the motor shaft bearing.

Figure 2: Permanent magnet stator. The two pieces establish a magnetic field.

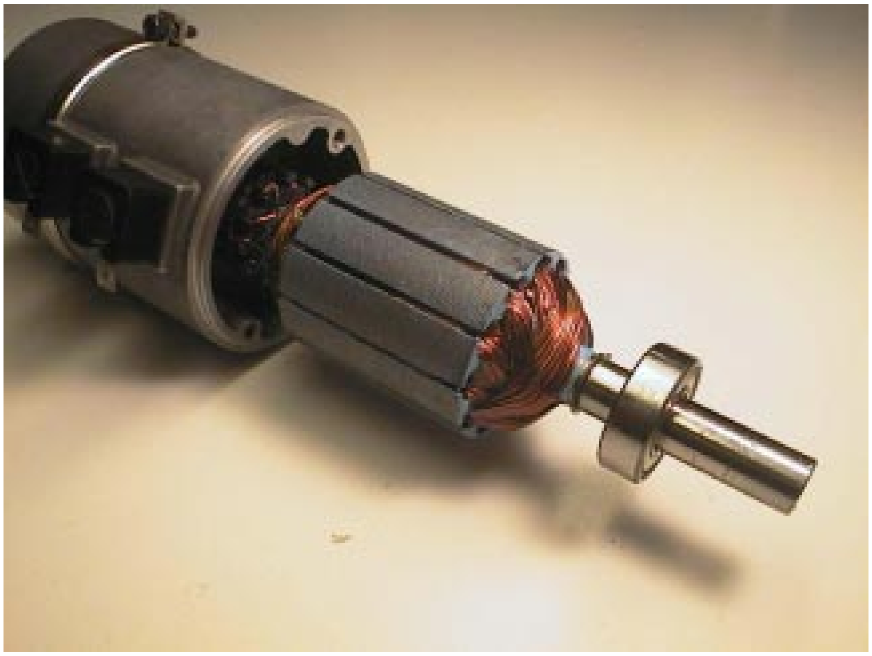

Figure 3: Coils of copper run lengthwise along the rotor and are called armature windings. These are responsible for carrying the current through the magnetic field of the stator.

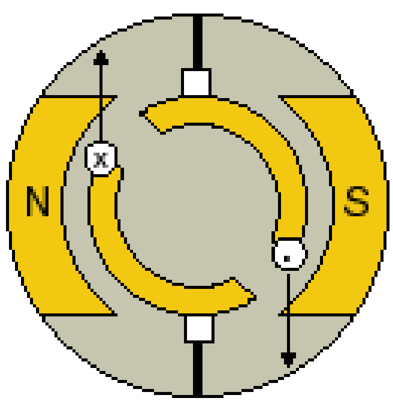

Figure 4: Simplified diagram of commutation.

Historically, the brushes were small pads of short metal wire but nowadays are just blocks of carbon. The brushes are connected to power and ground, and the commutator rings contact the brushes and transfer current to the correct armature windings. Figure 5 provides a rough picture of the brushes, commutator rings, and armature windings. To reverse the direction of motor rotation, the polarity of the applied voltage is changed. An important property of permanent magnet DC brush motors is that generated torque increases linearly with the increasing current through the armature windings, which makes it very easy to control.

3 PID Controllers

A control problem is generally defined as how to achieve the desired output from a physical system. For this purpose, automatic control systems are employed. The "intelligence" necessary to produce the desired output is the responsibility of the controller. The "intelligence" appears in the form of an algorithm, commonly referred to as a control law. The most common control used in industrial practice is PID control.[1]

In the discussion that follows, we will use a DC motor as the physical system to be controlled. Recall that the two first order differential equations that describes the relationship of angular velocity and angular position to input voltage are given by

$$\begin{aligned} \frac{d\omega}{dt} &= A\omega + B \cdot V \\ \frac{d\theta}{dt} &= \omega \end{aligned}$$

where ω is the angular velocity of the motor shaft, θ is the angular position, t represents time, A and B are experimental constants, and V is the input voltage. As far as numerical values are concerned, in future simulations, we will use $A = -0.1$ and $B = 100$. It is also important to note that both position or velocity can be considered as the output which we wish to control. We will never seek to control both at the same time because this is either redundant (desired position and velocity are both zero) or impossible.

3.1 Proportional Control

Given the first order differential equation relating voltage V to velocity ω for a DC motor,

$$\dot{\omega} = A\omega + B \cdot V$$

the control problem is how to achieve a desired rotational velocity ω_{des}. One possibility would be to input set amounts of voltage and to take measurements on how fast the motor turned in each case. We could then tabulate the experimental input/output relationship. Given a desired rotational velocity we would look at our table and determine the voltage necessary to attain this. Alternatively, by experimentally determining the values of A and B we could also calculate the necessary voltage to produce the desired rotational velocity through our knowledge of differential equations.

Both methods are examples of "open-loop" control in that in both cases, the actual output (ω) of the motor is not used in determining the input voltage. This causes a serious problem when we try

[1] PID stands for [P]roportional-[I]ntegral-[D]erivative.

Figure 5: A blurry picture of the workings of the motor. A brush (carbon block) contacts a commutator ring which routes current to the correct armature winding.

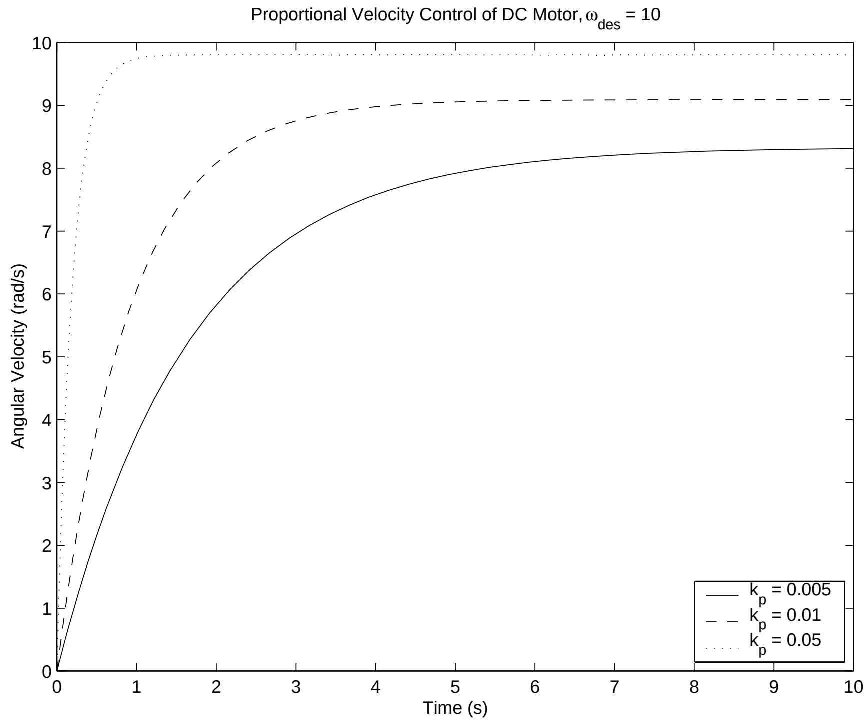

Figure 6: Proportional velocity control results.

to implement open-loop control in the real world. For example, what happens if over time the motor shaft bearings become stiffer, effectively changing the A parameter of our model? What if a load torque is applied to the motor shaft?

A better alternative is to determine the voltage input based upon the desired performance and actual performance. This is referred to as "closed-loop" control. Define the velocity error e such that

$$e = \omega_{des} - \omega$$

Now make the input voltage proportional to the velocity error:

$$V = k_p e$$

where k_p is a constant value. Intuitively, the operation of proportional controllers are easy to understand. Take the initial condition that $\omega = 0 rad/s$ and $\omega_{des} = 10 rad/s$. The initial value of rotational error would be $10 rad/s$ and after being multiplied by a constant gain, the input voltage would be determined. The motor immediately starts accelerating.

As the rotational velocity approaches the desired value, it can be seen that the velocity error decreases, reducing the input voltage, which reduces the acceleration of the motor. Over time, the velocity error reaches a finitely small value. The performance of our controller is given in Figure 6. For velocity control, a higher proportional gain leads to a faster response.

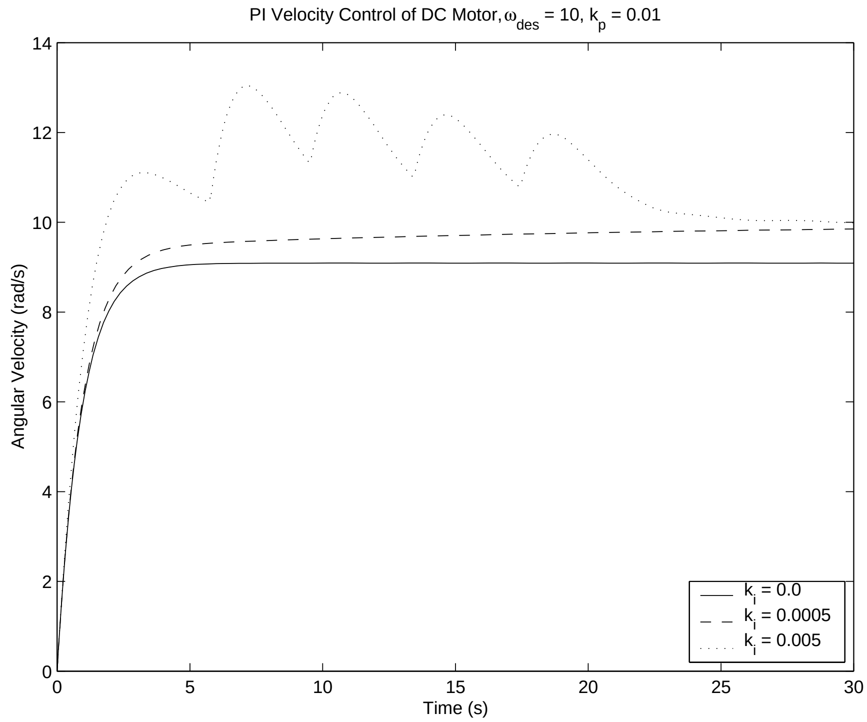

Figure 7: PI velocity control results.

3.2 Steady-State Error and the Use of Integral Control

Given the differential equation above, it can be shown that for a finite value of proportional gain, there will be a steady-state error in the velocity tracking. Clearly this is shown in Figure 6. Intuitively it can be argued that if there is no velocity error, then input voltage is equal to zero, implying that the motor decelerates! The amount of steady state error is dependent upon the value of the proportional gain constant. For high gains, steady-state error is reduced.

However, high gain controllers are sometimes undesirable, and a different method for eliminating steady state error is used. Amend the control law such that

$$V = k_p e + k_i \int_0^t e\,dt$$

Notice that the control signal is based upon current error information but also past error information through the use of the integral term. It can be shown that the addition of the integral component eliminates steady-state error in this system as shown in Figure 7. Integral control can be dangerous, however, as evidenced in Figure 7 by the choice of $k_i = 0.005$. Moderate integral gains can lead to system instability.

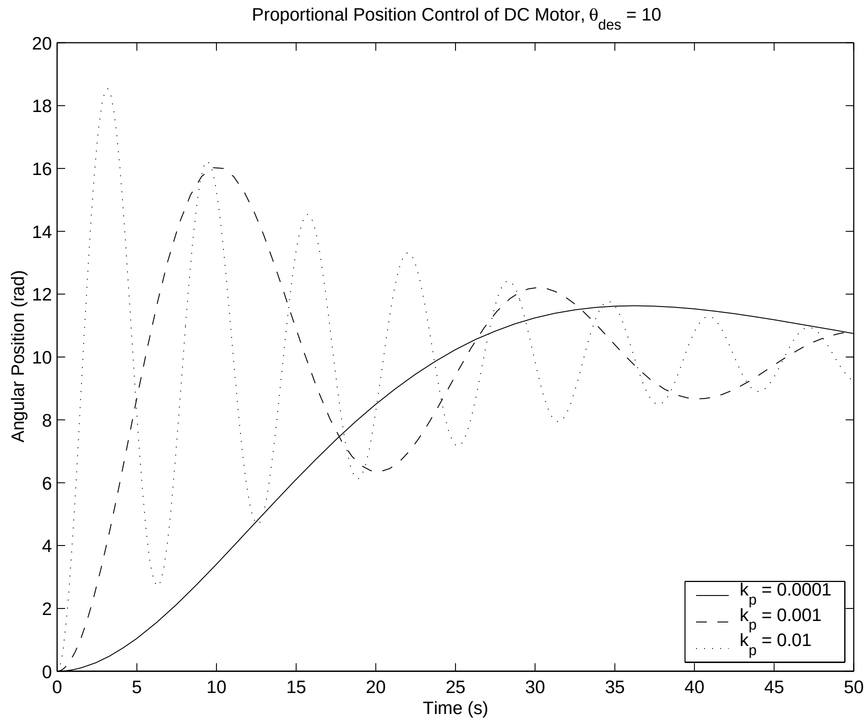

Figure 8: Proportional position control results.

3.3 Overshoot, Oscillation, and the Use of Derivative Gain

High gain controllers benefit from fast response times and high frequency tracking. The downside of a high gain controller is that the system often exhibits overshoot to a step input, and oscillation during tracking. To demonstrate this, consider the problem of position control. Similar to velocity control, let the desired performance be specified as θ_{des}, and the error be defined as $e = \theta_{des} - \theta$.

Using proportional control the response of the DC motor to a step input in desired position is given in Figure 8. Note that using moderate gains causes extreme overshoot and oscillation in the actual response of the motor. Lowering the gain attenuates these problems but increases the response time. Realize that integral control is not a solution to this problem — integral control increases this instability!

A method for increasing the effective damping of the system is to once again amend the control law such that

$$V = k_p e + k_i \int_0^t e\, dt + k_d \frac{de}{dt}$$

Intuitively, while the integral term gives past error information, the addition of the derivative term effectively acts to give future information. Derivative gain is used to smooth the response of the system and this is shown in Figure 9. It's important to realize, however, that implementing derivative control is usually very difficult. This stems from the fact that numerical differentiation is often used, and this procedure tends to amplify noise in the position measurement.

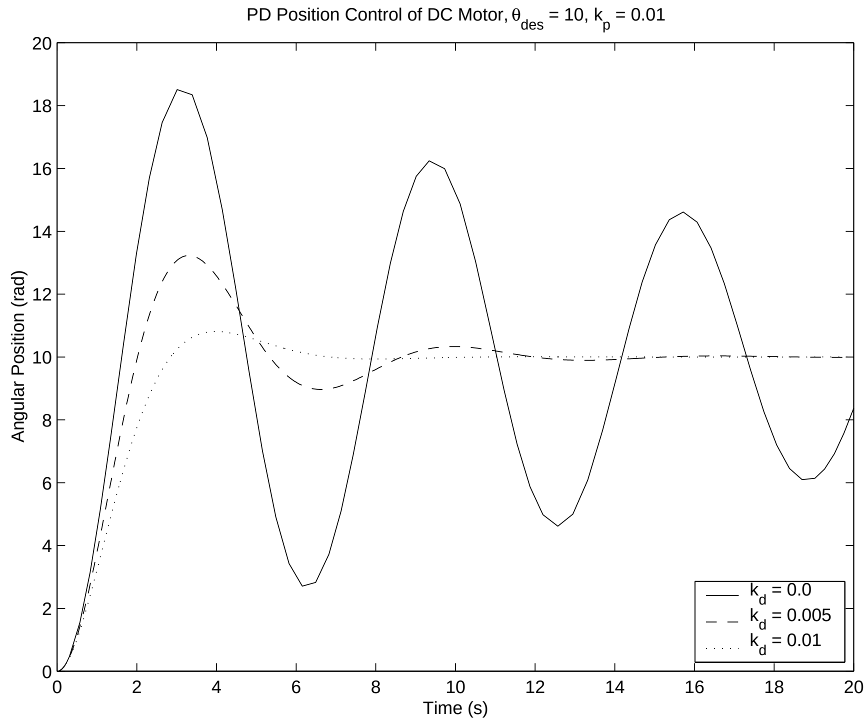

Figure 9: PD position control results.

3.4 Tuning PID Controllers

The act of selecting k_p, k_i, and k_d for desired system performance is called gain tuning. The vast majority of the control effort should be supplied by the proportional component, while small amounts of integral and derivative control are added to address the specific problems described above. Start by just using proportional control to achieve roughly the desired transient response. Then add integral and derivative control in moderation if the system behavior requires it.